South East Asian J. of Mathematics and Mathematical Sciences
Vol. 19, No. 3 (2023), pp. 87-100

DOI: 10.56827/SEAJMMS.2023.1903.9 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

CHARACTERISATION OF EQUIVALENT NORMS ON A LINEAR SPACE USING EXPONENTIAL VECTOR SPACE

Dhruba Prakash Biswas, Priti Sharma* and Sandip Jana

Department of Pure Mathematics,
University of Calcutta,
35, Ballygunge Circular Road, Kolkata-700019, INDIA

E-mail : dhrubaprakash28@gmail.com, sjpm@caluniv.ac.in

*Bangabasi College,
19, Rajkumar Chakraborty Sarani, Kolkata - 700009, INDIA

E-mail : mspriti23@gmail.com

(**Received:** *Aug. 03, 2023* **Accepted:** *Dec. 24, 2023* **Published:** *Dec. 30, 2023)*

Abstract: In this paper we have found a necessary and sufficient condition for equivalence of two norms on a linear space using the theory of exponential vector space. Exponential vector space ('evs' in short) is an ordered algebraic structure which can be considered as an algebraic ordered extension of vector space. This structure is axiomatised on the basis of the intrinsic properties of the hyperspace $\mathscr{C}(\mathcal{X})$ comprising all nonempty compact subsets of a Hausdorff topological vector space $\mathcal{X}$. Exponential vector space is a conglomeration of a semigroup structure, a scalar multiplication and a compatible partial order. We have shown that the collection of all norms defined on a linear space, together with the constant function zero, forms a topological exponential vector space. Then using the concept of comparing function (a concept defined on a topological exponential vector space) we have proved the aforesaid necessary and sufficient condition; also we have proved using comparing function that in an infinite dimensional linear space there are uncountably many non-equivalent norms.

Keywords and Phrases: Equivalence of norms, exponential vector space, topological exponential vector space, zero primitive evs, comparing function.

2020 Mathematics Subject Classification: 46A99, 46B99, 06F99.